Kalman Filters For Noise Reduction

APL ASPIRE Program Student Showcase

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Air and Missile Defense Sector

May 11th, 2021

Johns Hopkins
Applied Physics Laboratory

The impact of Kalman filtering on all areas of applied mathematics, engineering, and sciences has been tremendous.

- Rudolf E. Kálmán

Background

Keshav Ganapathy

- Senior at Centennial High School in Ellicott City, Maryland
- I intend to pursue a a double major in computer science and mathematics

Internship

- ASPIRE Program at Johns Hopkins University Applied Physics Laboratory

Mentor

- Joshua McClellan is my mentor and he is a software engineer at APL.

Introduction

- In everyday life we are dependant on sensors
- These sensors often have slight inaccuracy caused my external an internal factors called **noise**
- In this study, we use two filtering techniques the Kalman Filter and Moving Average filter and compare the results of both algorithms

Where are Kalman Filters Used?

Kalman Filters can be applied to anything with a sensor. Tasks include GPS navigation, signal processing, predicting trajectories of objects and more!

Moving Average Filter

- A filter that works by creating a series of averages of subsets of the full data set.

Math

This can be mathematically represented with the equation below:

$$y[i] = \frac{1}{N}\sum_{j=0}^{N-1} x[i+j]$$

Moving Average Filter Example

- Take the example list: [45, 24, 49, 29, 16, 39, 45, 34, 30, 2] with N=2
- The first element can be represented by $\frac{(45+24)}{2} = 34.5$, the second by $\frac{(24+49)}{2} = 36.5$, etc.
- We arrive at the final filtered list of: [34.5, 36.5, 39 , 22.5, 27.5, 42, 39.5, 32, 16]

Kalman Filter

- The Kalman Filter is a Linear Quadratic Estimation algorithm
- Uses various noisy measurements to produce estimates that are often more accurate.
- Has a prediction and update step
- In the prediction step, the Kalman filter uses the noisy measurements to create estimates, and in the update step the estimates are updated using a weighted average.

Kalman Filter

The formulas used in both steps and their derivations can be found visually in [1] and mathematically in [2].

Prediction Step

$$\hat{\vec{\mathbf{x}}}_k = \mathbf{F}_k \hat{\vec{\mathbf{x}}}_{k-1}$$

$$\mathbf{P}_k = \mathbf{F}_k \mathbf{P}_{k-1} \mathbf{F}_k^T + \mathbf{Q}_k * a$$

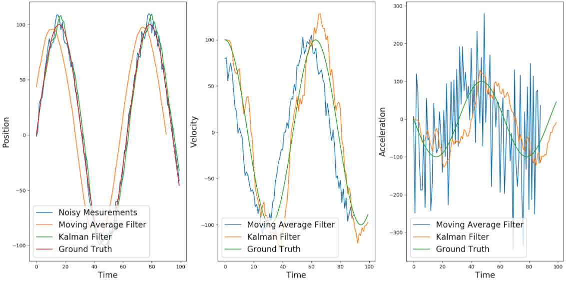
Update Step

$$\mathbf{K}' = \mathbf{P}_k \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_k \mathbf{H}_k^T + \mathbf{R}_k)^{-1}$$

$$\hat{\vec{\mathbf{x}}}'_k = \hat{\vec{\mathbf{x}}}_{\mathbf{k}} + \mathbf{K}'(\vec{\mathbf{z}}_k - \mathbf{H}_k \hat{\vec{\mathbf{x}}}_k)$$

$$\mathbf{P}'_k = \mathbf{P}_k - \mathbf{K}' \mathbf{H}_k \mathbf{P}_k$$

Results


Position
Time
Noisy Mesurements
Moving Average Filter
Kalman Filter
Ground Truth
Velocity
Time
Moving Average Filter
Kalman Filter
Ground Truth
Acceleration
Time
Moving Average Filter
Kalman Filter
Ground Truth

Quantifying Results

- To measure the performance of the two filters we took the mean of the absolute value of the residuals, M_r.

Math

This can be mathematically represented with the equation below:

$$M_r = avg(|predicted - true|)$$

Table of Results

Filter and Variable	Trial 1	Trial 2	Trial 3	Overall Average
Kalman Position	5.318	5.760	5.837	5.638
Moving Average Position	27.660	27.378	27.284	27.441
Kalman Velocity	15.845	15.515	14.307	15.222
Moving Average Velocity	32.495	32.041	32.405	32.314
Kalman Acceleration	42.464	41.346	40.800	41.537
Moving Average Acceleration	91.385	95.461	81.952	89.601

Conclusion

- As hypothesized, the Kalman filter significantly out performed the Moving Average filter
- Future work contains expanding functionality of the Kalman filter to handle multiple dimension

Questions?

Thank You for Listening!

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